Exam 2 Practice Problems

1. For the following matrices, find the characteristic polynomial, the eigenvalues (if they exist) and determine if the matrix is diagonalizable. If the matrix is diagonalizable, find the diagonalizing matrix and the resulting diagonal matrix.

$$A = \begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}.$$

2. For the matrices $A = \begin{bmatrix} 1 & 4 \\ 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, calculate e^A, e^B and find formulas for A^n, B^n, for all $n \geq 1$.

3. Solve the recurrence relation given by the equation $a_{k+2} = -2a_k + 3a_{k+1}$, with initial conditions $a_0 = 2$ and $a_1 = 4$.

4. Solve the following system of linear differential equations with the given initial conditions:

$$\begin{aligned} x_1'(t) &= 6x_1(t) - x_2(t) \\ x_2'(t) &= 2x_1(t) + 3x_2(t), \end{aligned}$$

with $x_1(0) = -1$ and $x_2(t) = 2$.

5. Determine if the vector $w = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$ is in the subspace of $\mathbb{R}^3$ spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$. If w is in the space spanned by v_1, v_2, v_3, write w as a linear combination of these vectors.

6. Determine whether or not the vectors v_1, v_2, v_3 in the previous problem are linearly independent.

7. Find a basis for the subspace of $\mathbb{R}^4$ spanned by $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ -4 \\ -2 \\ 0 \end{bmatrix}$.

8. Find the dimensions of the all of eigenspaces of the matrices in problem 1.